"SINGLE CHANNEL NON-STATIONARY STOCHASTIC SIGNAL SEPARATION USING LINEAR TIME-VARYING FILTERS"

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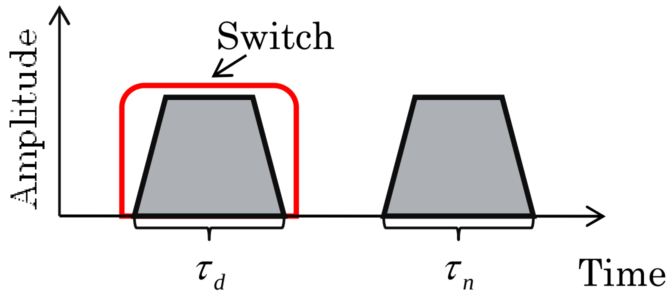
DEFINITIONS

- Desired signal $d(t)$
- Additive noise signal $n(t)$
- Observation $x(t) = d(t) + n(t)$
- Desired signal estimation $\hat{d}(t)$

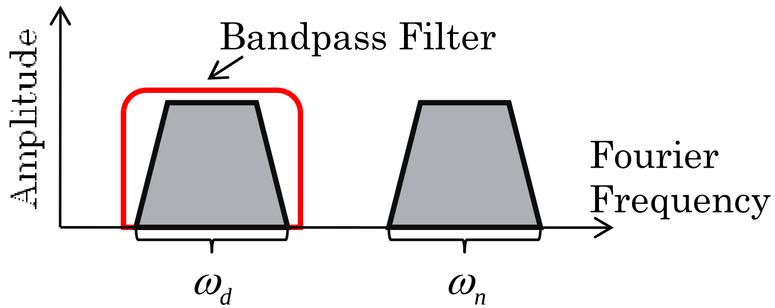
INTRODUCTION

- Signal separation techniques
  - Signals do not overlap in time domain



LTV filter

  - Non-overlapping power spectra

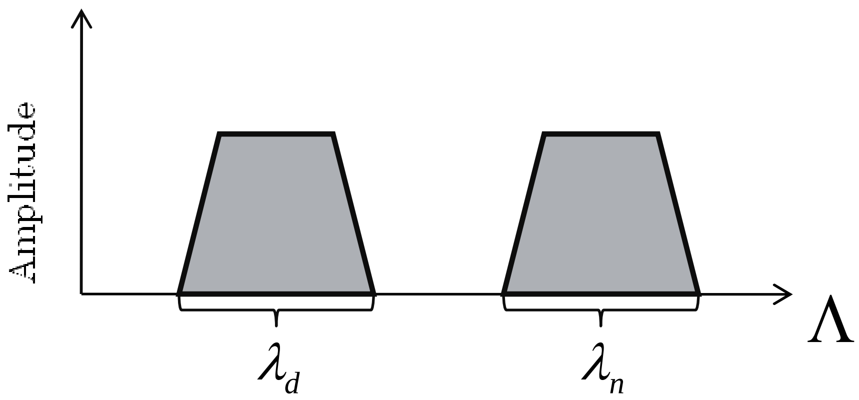


LTI filter

INTRODUCTION (2)

- Suggestion:

  Existence of arbitrary domain where the representations of the signals are disjoint



SIGNAL SEPARATION USING LTV FILTERS

- Separation by LTV filtration:

$$\hat{d}(t) = \int_{\mathcal{T}} h(t,\alpha)\, x(\alpha)\, d\alpha, \quad \forall t \in T$$

- Perfect separation of stochastic signals is achieved when:

$$\sigma^2(t) \triangleq E\left\{\left|\hat{d}(t) - d(t)\right|\right\} = 0$$

SIGNAL SEPARATION USING LTV FILTERS (2)

- Non-stationary Wiener-Hopf Filter (WHF):

$$R_{dx}(t,\tau) = \int_{\mathcal{T}} h(t,\alpha) R_{xx}(\alpha,\tau)\, d\alpha$$

with MSE given by:

$$\sigma^2(t) = R_{dd}(t,t) - \int_{\mathcal{T}} h(t,\alpha) R_{dx}(t,\alpha)\, d\alpha$$

- For "perfect separation":

$$R_{dd}(t,t) = \int_{\mathcal{T}} h(t,\alpha) R_{dx}(t,\alpha)\, d\alpha$$

POWER SPECTRA FOR NON-STATIONARY SIGNALS

- Spectral transforms

$$\boxed{\begin{aligned} X(\lambda) &= \int_{\mathcal{T}} x(t) K(t,\lambda)\, dt, \quad \forall \lambda \in \Lambda \\ \hat{x}(t) &= \int_{\Lambda} X(\lambda) k(\lambda,t)\, d\lambda, \quad \forall t \in T \end{aligned}}$$

  - Direct transform basis kernel $K(t,\lambda)$
  - Inverse transform basis kernel $k(\lambda,t)$

- For stochastic signals: $E\left\{\left|\hat{x}(t) - x(t)\right|^2\right\} = 0$

POWER SPECTRA FOR NON-STATIONARY SIGNALS (3)

- Definition: Generalized Power Spectrum

$$\boxed{\begin{aligned} P_{xx}\left(\lambda,\hat{\lambda}\right) &= \iint_{\mathcal{T}^2} R_{xx}\left(t,\tau\right) K\left(t,\lambda\right) K^*\left(\tau,\hat{\lambda}\right) dt d\tau \\ R_{xx}\left(t,\tau\right) &= \iint_{\Lambda^2} P_{xx}\left(\lambda,\hat{\lambda}\right) k\left(\lambda,t\right) k^*\left(\hat{\lambda},\tau\right) d\lambda d\hat{\lambda} \end{aligned}}$$

where $P_{xx}\left(\lambda,\hat{\lambda}\right) \triangleq E\left\{X\left(\lambda\right) X^*\left(\hat{\lambda}\right)\right\}$.

POWER SPECTRA FOR NON-STATIONARY SIGNALS (4)

- Definition: Generalized **Cross** Power Spectrum

$$
\boxed{
\begin{aligned}
P_{yx}\left(\lambda,\hat{\lambda}\right) &= \iint_{\mathcal{T}^2} R_{yx}\left(t,\tau\right) K\left(t,\lambda\right) K^*\left(\tau,\hat{\lambda}\right) dt d\tau \\
R_{yx}\left(t,\tau\right) &= \iint_{\Lambda_0^2} P_{YX}\left(\lambda,\hat{\lambda}\right) k\left(\lambda,t\right) k^*\left(\hat{\lambda},\tau\right) d\lambda d\hat{\lambda}
\end{aligned}
}
$$

TRANSFER FUNCTIONS

- Input-output relationship for LTV system:

$$y(t) = \int_T h(t,\tau)\,x(\tau)\,d\tau, \quad \forall t \in T$$

- Transfer function of LTV system:

$$h(t,\tau) = \iint_{\Lambda^2} H\left(\lambda,\hat{\lambda}\right) k(\lambda,t)\, K\left(\tau,\hat{\lambda}\right) d\lambda\, d\hat{\lambda}$$

$$H\left(\lambda,\hat{\lambda}\right) = \iint_{T^2} h(t,\tau)\, K(\tau,\lambda)\, k\left(\hat{\lambda},t\right) dt\, d\tau$$

$$Y(\lambda) = \int_\Lambda H\left(\lambda,\hat{\lambda}\right) X\left(\hat{\lambda}\right) d\hat{\lambda}$$

SEPARATION TECHNIQUES

- Definition: Ideal filter over the Λ-domain

$$Y(\lambda) = \begin{cases} 0, & \lambda \in \Lambda_H \\ X(\lambda), & \lambda \notin \Lambda_H \end{cases}$$

- Theorem: an ideal filter over the Λ-domain with impulse response $h(t,\tau)$ may be represented as

$$h(t,\tau) = \int_{\Lambda_H} k(\lambda,t) K(\tau,\lambda) d\lambda, \quad (t,\tau) \in T^2$$

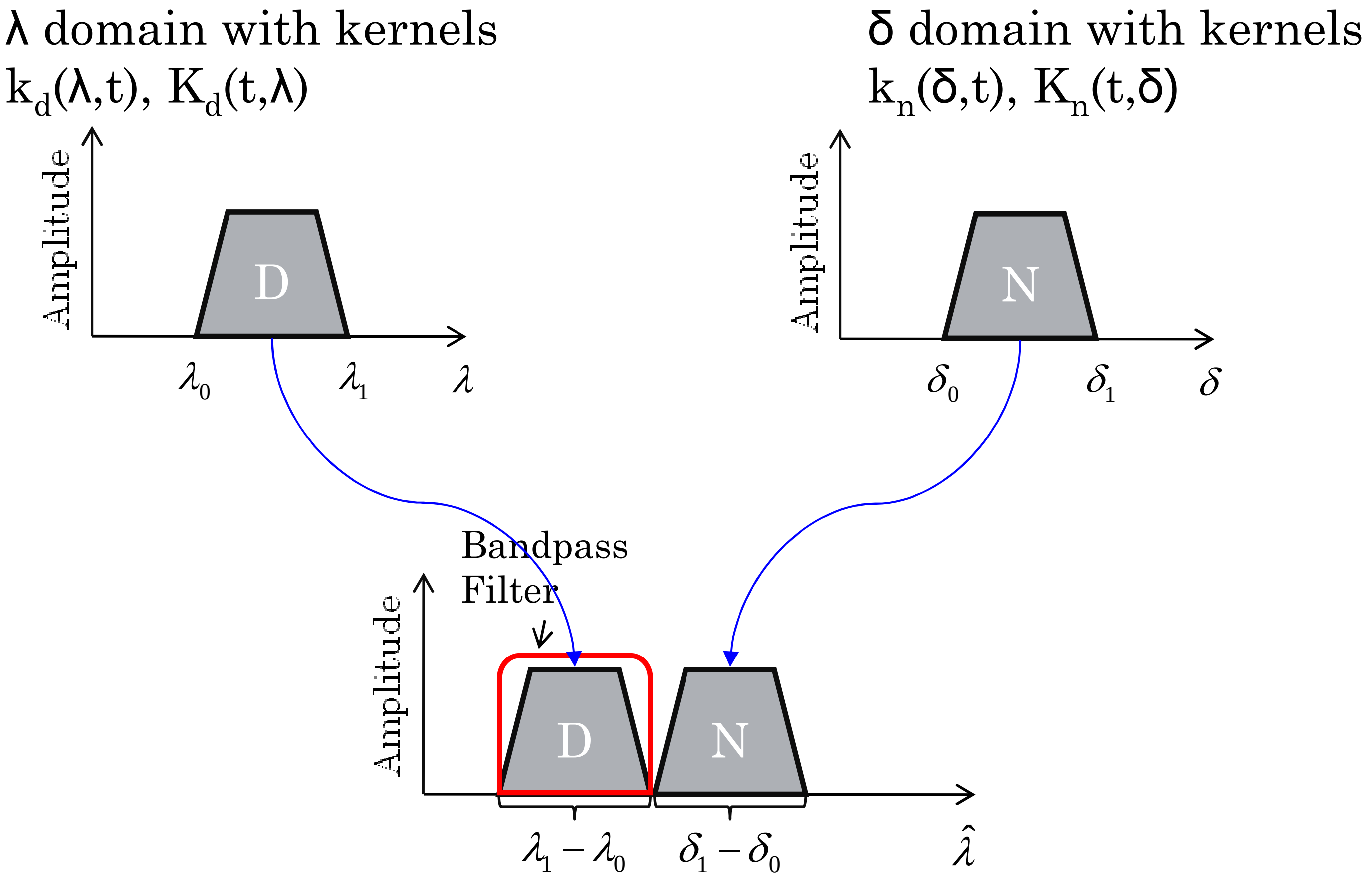
SEPARATION TECHNIQUES (2)

- Theorem (Ideal Filter Component):

  the WHF $h(t,\tau)$, which give perfect separation, is an ideal filter $H\left(\lambda,\hat{\lambda}\right)$ assuming that the signals have disjoint generalized spectrum components.

- How do we chose the transform kernels?

SELECTING TRANSFORM KERNELS



SELECTING TRANSFORM KERNELS (2)

- The kernel for this combined domain:

$$k\left(\hat{\lambda},t\right)=\begin{cases} k_n\left(\lambda\left(\hat{\lambda}\right),t\right), & \left\{\hat{\lambda}:\left|N\left(\lambda\left(\hat{\lambda}\right)\right)\right|\geq 0\right\} \\ k_d\left(\delta\left(\hat{\lambda}\right),t\right), & \left\{\hat{\lambda}:\left|D\left(\delta\left(\hat{\lambda}\right)\right)\right|\geq 0\right\} \end{cases}$$

when

$$\left\{\hat{\lambda}:\left|N\left(\lambda\left(\hat{\lambda}\right)\right)\right|\geq 0\right\}\cap\left\{\hat{\lambda}:\left|D\left(\delta\left(\hat{\lambda}\right)\right)\right|\geq 0\right\}=\varnothing$$

SELECTING TRANSFORM KERNELS (3)

- The *concatenated kernel*:

$$k(\lambda,t)=\sum_i 1_{\Lambda_i}(\lambda)k_i'(\lambda,t)$$

where $i\in\{d,n,v\}$, $1_{\Lambda_i}(\lambda)=\begin{cases}1, & \lambda\in\Lambda_i\\ 0 & \lambda\notin\Lambda_i\end{cases}$ and

Λ_v is "unused" space: $\Lambda=\Lambda_d\oplus\Lambda_n\oplus\Lambda_v$.

- Such a domain exists when the transform is isomorphic.

DISCRETE CASE

- Discrete time and discrete spectral domains

$$X(p) = \sum_{n \in N} x(n) K(n,p)$$

$$x(n) = \sum_{p \in P} X(p) k(p,n)$$

- For non-degenerate transform:

$$\sum_{n \in N} k(p,n) K(n,\hat{p}) = \delta(p - \hat{p})$$

$$\sum_{p \in P} K(n,p) k(p,\hat{n}) = \delta(n - \hat{n})$$

DISCRETE CASE (2)

- Defining $\quad [\underline{x}]_n = x(n), \qquad [\underline{X}]_p = X(p)$

  $[\underline{\underline{k}}]_{pn} = k(p,n), \qquad [\underline{\underline{K}}]_{np} = K(n,p)$

  we can write compactly

$$\underline{X} = \underline{\underline{K}}^T \underline{x} \quad , \quad \underline{x} = \underline{\underline{k}}^T \underline{X} \quad , \quad \underline{\underline{k}} \cdot \underline{\underline{K}} = \underline{\underline{K}} \cdot \underline{\underline{k}} = \underline{\underline{I}}$$

- Concatenating discrete spectra:

$$\underline{\underline{k}} \cdot \underline{\underline{K}} = \begin{bmatrix} k_d \\ \hline k_n \\ \hline k_v \end{bmatrix} \left[K_d \,|\, K_n \,|\, K_v \right] = \underline{\underline{I}}$$

SEPARATING FILTERED MODULATED SIGNALS

- General form:

$$d(t) = \int_T h_d(t,\tau)\, a(\tau)\, d\tau$$

$$n(t) = \int_T h_n(t,\tau)\, b(\tau)\, d\tau$$

where:

$h_d(t,\tau),\ h_n(t,\tau)$ known deterministic signals

$a(t),\ b(t)$ bandlimited to $\pm\omega_c$, unknown

SEPARATING FILTERED MODULATED SIGNALS (2)

- Using the described method we define:

$$k(\lambda,t)=\begin{cases}\int_T h_d(t,\tau)e^{j(\lambda-\omega_c)\tau}d\tau, & \omega_c\in[0,2\omega_c)\\ \int_T h_n(t,\tau)e^{j(\lambda+\omega_c)\tau}d\tau, & \omega_c\in[-2\omega_c,0)\\ k_v(\lambda,t), & else\end{cases}$$

- The signal are separable if the transform is isomorphic.

DISCRETE MODULATION

- Following the continuous case, define:

$$\left[\hat{k}_d\right]_{pn} = W_N^{n(p-q_c)}, \quad p \in P_D$$

$$\left[\hat{k}_v\right]_{pn} = k_v(p,n), \quad p \in P_V \qquad \left(W_N^{np} = (1/N)e^{jnp(2\pi/N)}\right)$$

$$\left[\hat{k}_n\right]_{pn} = W_N^{n(p+q_c)}, \quad p \in P_N$$

then: $$\boxed{k^T = \left[H_d \hat{k}_d^T \mid H_n \hat{k}_n^T \mid \hat{k}_v^T\right]}$$

where $H_d = diag\left(h_d(t)\right), \qquad H_n = diag\left(h_n(t)\right),$

$$\left[k_v\right]_{pt} = W_N^{tp}$$

DISCRETE MODULATION (2)

- Calculating Ideal Filter Matrix:

if rank(k)=N *then*

Generalized Bandpass Filter $\boxed{H = diag\left[\underline{1}_{2p_c} \mid \underline{0}_{N-2p_c}\right]}$

Ideal Filter Matrix $\boxed{A = k^T HK^T}$

else

Signals are inseparable

endif

DISCRETE MODULATION (3)

- Noise gain for Filter Matrix:

  in the case of white noise in the input of the filter, the noise gain is given by

$$\eta_{WGN} \triangleq \frac{E\left\{\underline{y}\,\underline{y}^T\right\}}{E\left\{\underline{w}\,\underline{w}^T\right\}} = \frac{1}{N} trace\left[AA^T\right]$$

  where $\underline{y}$ is the output.

UNIFORM MODULATION

- Uniformly Modulated Process:

$$y(t) = c(t)x(t), \quad t \in T$$

, x(t) – stochastic process, c(t) – known signal.

Then:

$$d(t) = h_d(t)a(\tau)$$
$$n(t) = h_n(t)b(\tau)$$

UNIFORM MODULATION – Chirp-Modulated Signals

- Modulating signals:

$$h_d(t) = \cos\left(2\pi f_i \frac{t}{f_s}\left(1 + \frac{1}{\tau_d}\frac{f_f - f_i}{f_i}\frac{t}{f_s}\right)\right)$$

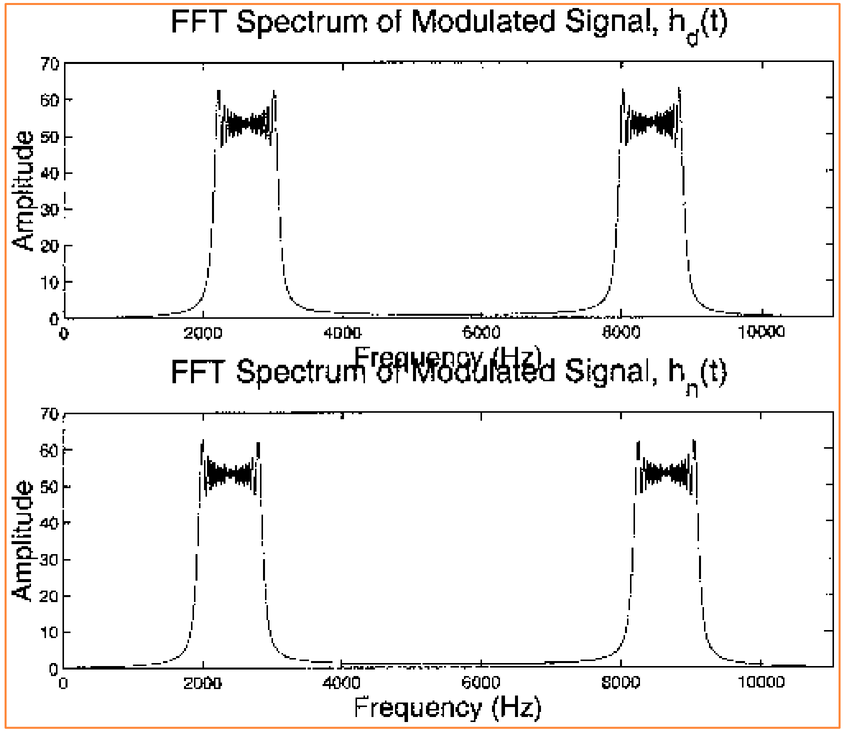
$$h_n(t) = \cos\left(2\pi f_i' \frac{t}{f_s}\left(1 + \frac{1}{\tau_d}\frac{f_f' - f_i'}{f_i'}\frac{t}{f_s}\right)\right)$$

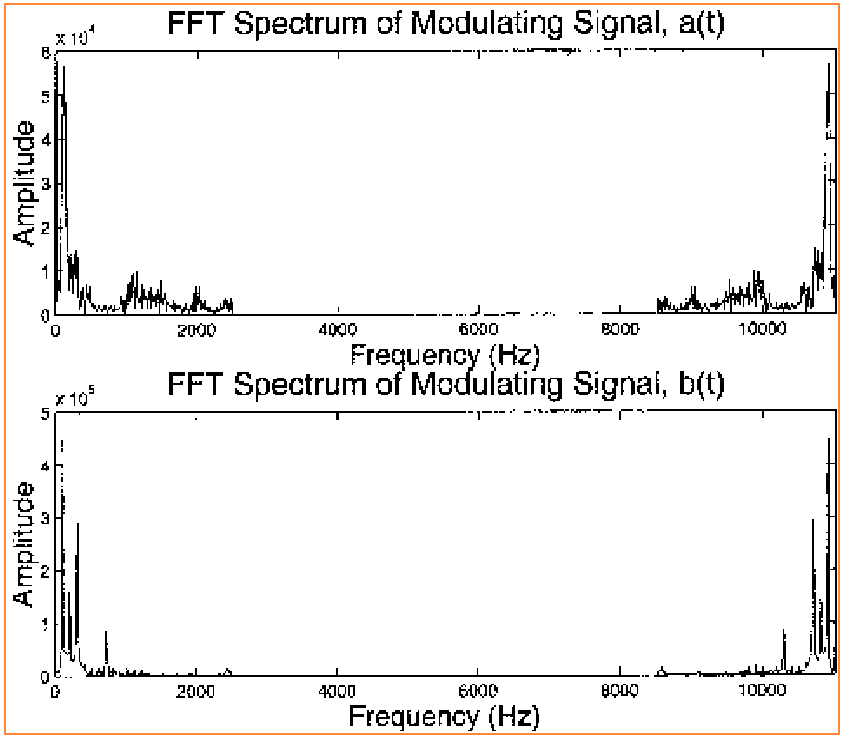
- For:

$$f_i = f_f' = 4250\,Hz, \quad f_f = f_i' = 5750\,Hz, \quad f_s = 22.05\,kHz,$$

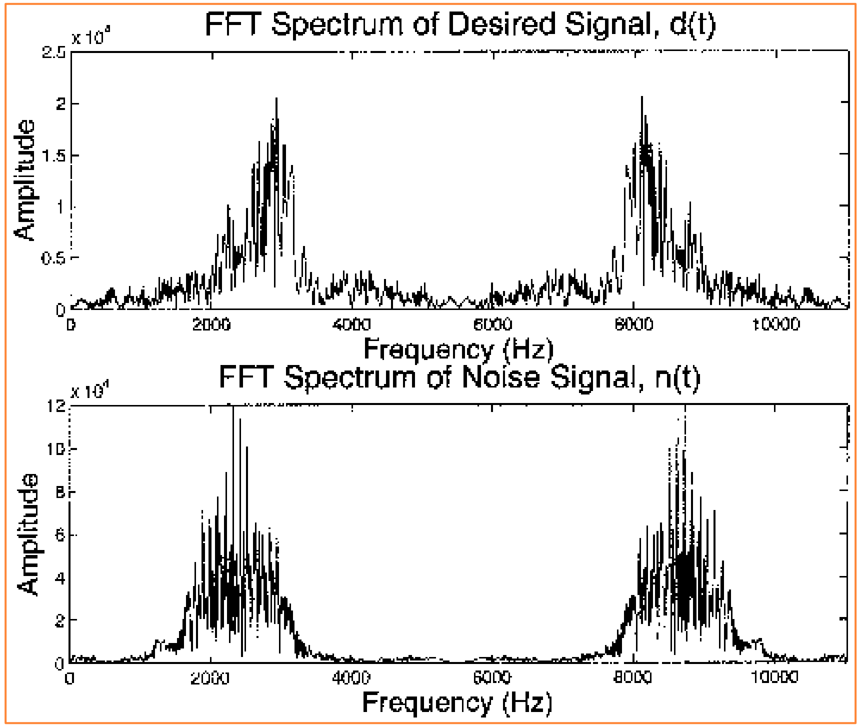
$$\tau_d = \tau_n = 70\,ms, \qquad f_c = 5\,kHz$$

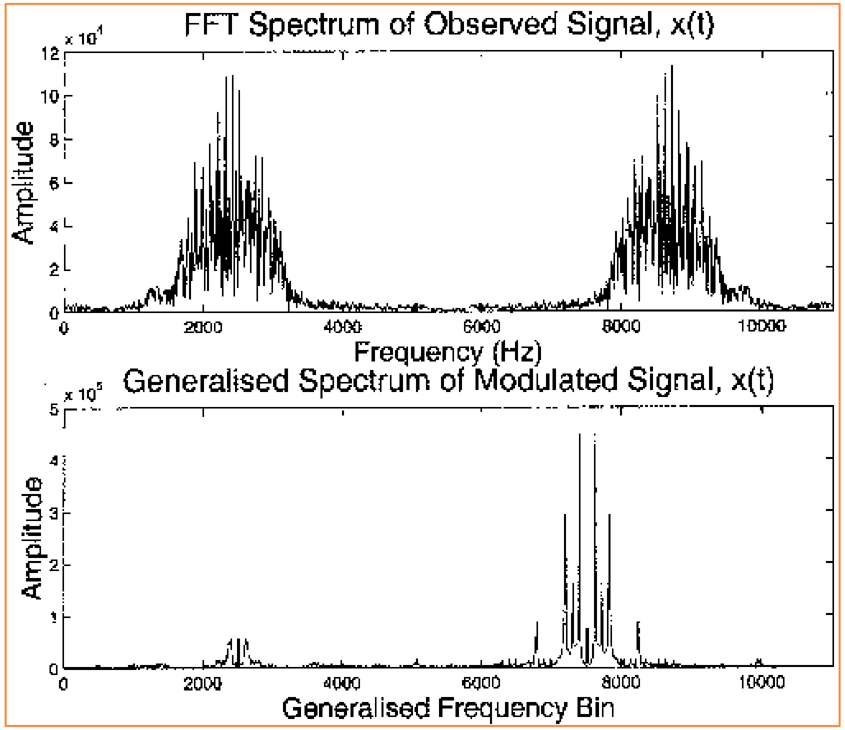
UNIFORM MODULATION – Chirp-Modulated Signals (2)



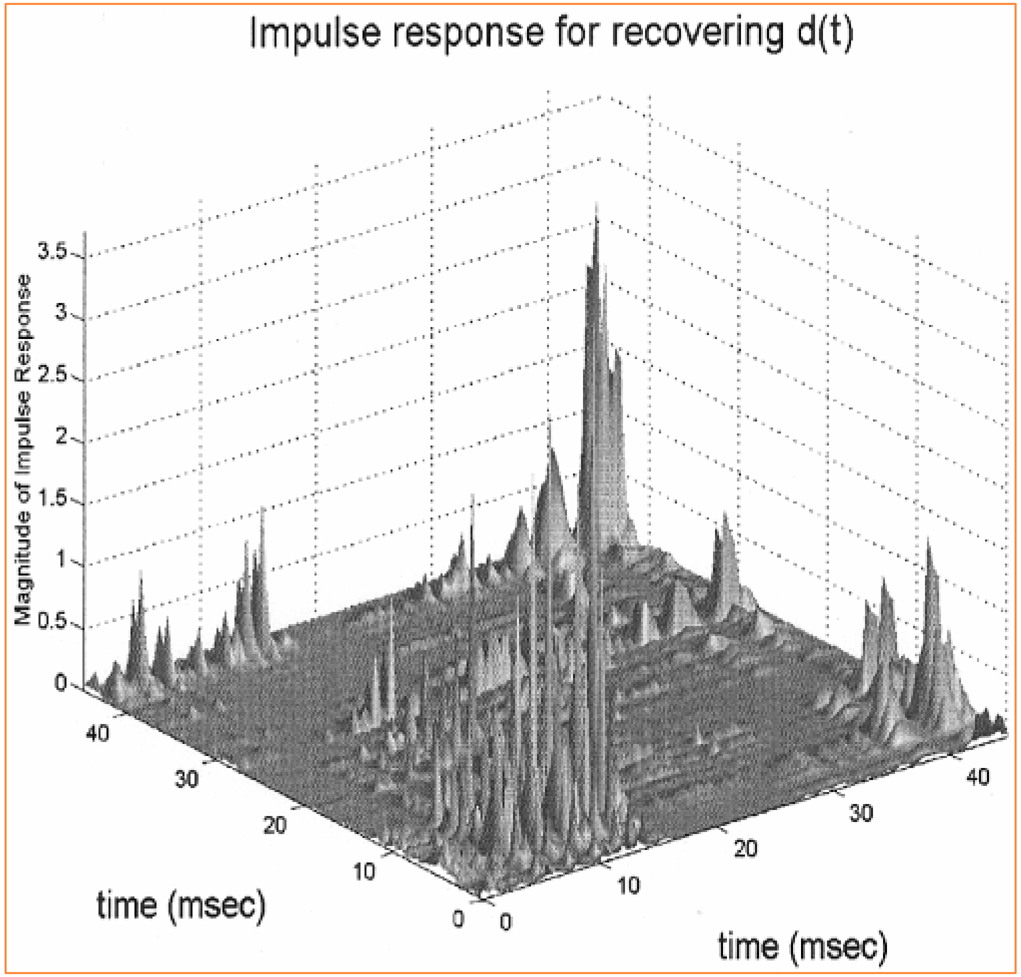


UNIFORM MODULATION – Chirp-Modulated Signals (3)





UNIFORM MODULATION – Chirp-Modulated Signals (4)



Conclusions

- Nonstationarity is useful when only a single observation of mixture of signals is available.
- Separation with one observation sensor can only be achieved by exploiting prior knowledge of signals structure.
- Signal separation technique by concatenating frequency domains was introduced and specifically applied to uniformly modulated signals.

Proposed Improvement – Separation of Chirp-Modulated signals

- The discussed algorithm uses Fourier Transform to build the *concatenated kernel* k, using all the given samples of x(t). These exist miltiple $h_d(t)$ and $h_n(t)$ that do not allow perfect separation.
- The improvement: use of STFT instead of Fourier Transform. By performing short time analysis we could achieve better results with the signals that are not separable using the discussed algorithm.